A Cell Method Formulation of Three Dimensional Electro-Thermo-Mechanical Contact Problems with Mortar Discretization

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Abstract — A three-dimensional mortar method for solving fully coupled electro-thermo-mechanical contact problems is presented. The formulation is based on the Cell Method which is founded on the duality principle. Contacting domains are linked together by introducing a new reference frame, i.e. the mortar surface. Field discontinuities across contact interfaces are simulated by suitable constitutive operators. Compatibility constraints are imposed by dual Lagrange multipliers defined on the mortar surface. Coupled non-linear algebraic equations are cast into a saddle point problem, which can be reduced to a positive definite system by exploiting duality. In such a way standard preconditioned iterative solvers can be used.

I. INTRODUCTION

Contact problems arise in wide variety of engineering applications such as metal forming, RSW, and connectors. The numerical solution of these problems is particularly complex due to the non-linear behavior of fully coupled PDEs. To date only a few methods encompassing multiple physics contact effects have been proposed [1][2]. Domain decomposition methods (DDMs) are effective when solving contact problems since the computational domain is split into several subdomains (e.g. contacting parts) coupled together by suitable projection operators. Most of DDMs are based, however, on FEM formulations which require a careful definition of trace operators and Sobolev spaces to ensure the convergence of numerical procedures [3]. The Cell Method (CM) offers a different perspective: equations are expressed directly in algebraic form by means of integral variables and discrete operators [4]. This feature makes the CM well suited for multiphysics problems [5].

A three-dimensional (3D) mortar cell method (MCM) for fully coupled electrical-thermal contact problems has been recently presented [6]. Its basic advantage compared to mortar FEMs is that the final system (in saddle point form) can be reduced to a positive definite system, which can be easily solved with efficient preconditioned iterative methods like PCG. The MCM is extended here to elasticity in order to simulate more realistic contact models. A static multibody frictionless contact with small displacements and linear elastic materials is analyzed. The following physics couplings are taken into account: (i) displacements are influenced by thermal field due to deformation stresses and Young's modulus; (ii) thermal generation is due to Joule heating depending on both temperature and electric scalar potential distributions; (iii) the electric conductivity decreases with temperature; (iv) electric and thermal contact conductivity distributions are pressure dependent.

II. MORTAR FORMULATION

The computational domain is depicted in Fig. 1: A two body contact problem is considered. Contacting bodies Ω_1, Ω_2 are meshed with non-conforming grids and continuity between subdomains is enforced by appropriate Lagrange multipliers, which are defined on the *mortar surface* Γ_m interleaved between contact interfaces Γ_{c1}, Γ_{c2} (Fig. 1). The bottom surface is constrained (Dirichlet BCs); Neumann conditions are applied on $\partial\Omega_1$ (**t**: traction field; **J**: electric current density; **q**: heat flux density). $\mathbf{f}_b$ are external body forces applied on the bulk domain $\Omega = \Omega_1 \cup \Omega_2$.

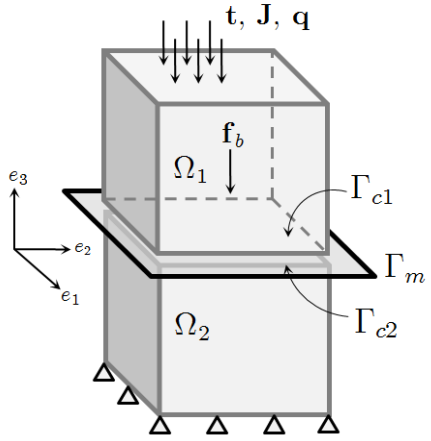

Fig. 1. Two body multiphysics contact problem.

A. Contact problem on the mortar surface

In discrete CM formulations problem variables are integral quantities (potentials, line integrals, and fluxes) defined on dual cell complexes. The duality principle (DP) is the basic idea of the CM: constitutive operators are mappings linking variables of different cell complexes. It is shown here that contact interfaces can be coupled symmetrically by the DP. The mortar surface as bulk domains is discretized in dual cell simplexes, where Lagrange multipliers (potentials and fluxes) are defined: electric potentials $v_m^{\pm}$, temperatures $\theta_m^{\pm}$, and displacements $u_m^{\pm}$ on primal nodes; currents j_m, heat fluxes q_m, and tractions t_m on dual faces. Degrees of freedom are projected together with geometric quantities: potentials are mapped from Γ_m to Γ_{c1} and Γ_{c2}; vice versa, fluxes are mapped from Γ_{c1} and Γ_{c2} to Γ_m. For the sake of brevity, in the following the electric coupling is presented only; thermal and mechanical problems are closely similar.

Kirchhoff's voltage law across the contact is imposed by means of the projection matrix P as:

$$[v_m] = P\, v_c \tag{1}$$

where $[v_m] = v_m^+ - v_m^-$ are potential jumps across Γ_m and v_c are potentials on contact interfaces, selected from the global array of potentials as $v_c = Q\, v$. Kirchhoff's current law is imposed by a dual projection operator $\widetilde{P} = P^{\mathrm{T}}$, as:

$$\widetilde{P}\, j_m + j_c = 0 \tag{2}$$

which makes it possible to obtain a symmetric coupling. Potential jumps across contact interfaces can be simulated by introducing appropriate constitutive relationships, e.g.

$$j_m = -M_{\sigma c}\, [v_m] \tag{3}$$

where the (diagonal) matrix $M_{\sigma c}$ is built by integrating the contact conductivity over Γ_m. Conductivity is estimated from statistical relationships depending on contact pressure, surface roughness and material hardness parameters [7].

In the mechanical problem displacements are assumed to be continuous across contact interfaces, i.e. $[u_m] = 0$.

B. Discrete problem in bulk domains

Degrees of freedom in the electric conduction problem are voltages e on primal edges and currents j on dual faces. Voltages are mapped to the dual complex by Ohm's law:

$$j = M_\sigma(\theta)\, e \tag{4}$$

where coefficients depends on temperature θ. Voltages are expressed in terms of potentials by Kirchhoff's voltage law $e = -G\, v$, where G is the gradient operator. The balance equation written for dual cells is Kirchhoff's current law:

$$\widetilde{D}\, j + j_c = j_s \tag{5}$$

where $\widetilde{D} = -G^{\mathrm{T}}$ is the divergence operator, j_s are source currents, and j_c are currents on Γ_{c1} and Γ_{c2}. Constitutive and continuity equations are assembled together as follows:

$$G^{\mathrm{T}} M_\sigma(\theta) G\, v + j_c = j_s \tag{6}$$

The thermal stiffness matrix can be assembled like (6) with a constant conductance operator instead:

$$G^{\mathrm{T}} M_\lambda G\, \theta + q_c = q_s + w(v, \theta) \tag{7}$$

where q_c are heat fluxes at contact interfaces, q_s are source fluxes and w are internal heat generations inside dual cells. The discrete formulation of elasticity is widely discussed in [8] and extended to coupled problems in [5]. By introducing covector-valued (stresses σ, tractions t) and vector-valued quantities (strains ϵ, displacements u) force equilibrium can be stated in the form of a discrete balance law:

$$\widetilde{D}\, \sigma + t_c = t_s + f_b + f_{th}(\theta) \tag{8}$$

where t_c are traction at contact interfaces, t_s are source tractions, f_b are external body forces, and f_{th} are thermal deformations. Strains are mapped to stresses by means of the elasticity tensor $M_e(\theta)$, which depends on temperature. The stiffness matrix is obtained as in the other problems:

$$G^{\mathrm{T}} M_e(\theta) G\, u + t_c = t_s + f_b + f_{th}(\theta) \tag{9}$$

C. Coupled problem in saddle point form

Using the proposed mortar approach the final non-linear system can be assembled into saddle point form, that is

$$\begin{pmatrix} G^{\mathrm{T}}M_\sigma G & \mathbb{O} & \mathbb{O} & -\Pi^{\mathrm{T}} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & G^{\mathrm{T}}M_\lambda G & \mathbb{O} & \mathbb{O} & -\Pi^{\mathrm{T}} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & G^{\mathrm{T}}M_e G & \mathbb{O} & \mathbb{O} & -\Pi^{\mathrm{T}} \\ \Pi & \mathbb{O} & \mathbb{O} & M_{\sigma c^{-1}} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \Pi & \mathbb{O} & \mathbb{O} & M_{\lambda c^{-1}} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \Pi & \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix} \begin{pmatrix} v \\ \theta \\ u \\ j_m \\ q_m \\ u_m \end{pmatrix} = \begin{pmatrix} j_s \\ q_s + w \\ t_s + f_b + f_{th} \\ 0 \\ w_m \\ 0 \end{pmatrix} \tag{10}$$

where $M_{\sigma c^{-1}}$ and $M_{\lambda c^{-1}}$ are diagonal resistance matrices, and $\Pi = PQ$ is a projection operator. The fully coupled system cannot be solved directly since it is badly scaled and conditioned. It can be reduced to a well conditioned system by eliminating Lagrange multipliers according to Schur's complement method. Moreover, displacements should lie in the kernel of the projection operator since mechanical field is assumed to be continuous across contact interfaces.

The solution strategy adopted for the non-linear system is discussed more deeply in the paper, where formulation and implementation details are provided. Numerical results are compared with those obtained by a commercial software package for analyzing multiphysics problems.

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